

# The $X(3872)$ and other possible $XYZ$ molecular states

P.G. Ortega, J. Segovia, D.R. Entem and F. Fernández

*Grupo de Física Nuclear y IUFFyM  
Universidad de Salamanca, 37008 Salamanca, Spain*

**Abstract.** We perform a coupled channel calculation of the  $DD^*$  and  $c\bar{c}$  sectors in the framework of a constituent quark model. The interaction for the  $DD^*$  states is obtained using the Resonant Group Method (RGM) and the underlying quark interaction model. The coupling with the two quark system is performed using the  $^3P_0$  model. The  $X(3872)$  is found as a molecular state with a sizable  $c\bar{c}$  component. A comparison with Belle and BaBar data has been done, finding a good agreement. Other possible molecular molecular states are discussed.

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## INTRODUCTION

In the last years a number of exciting discoveries of new hadron states, the so called  $XYZ$  mesons, have challenged our description of the hadron spectroscopy. Among them, one of the most mysterious states is the well established  $X(3872)$ . It was first discovered by the Belle Collaboration in the  $J/\psi\pi\pi$  invariant mass spectrum of the decay  $B^+ \rightarrow K^+\pi^+\pi^-J/\psi$  [1]. Its existence was soon confirmed by BaBar [2], CDF [3] and D0 [4] Collaborations. The world average mass is  $M_X = 3871.2 \pm 0.5\text{ MeV}$  and its width  $\Gamma_X < 2.3\text{ MeV}$ . The measurements of the  $X(3872) \rightarrow \gamma J/\psi$  decay [5, 6] implies an even  $C$ -parity. Moreover angular correlation between final state particles in the  $X(3872) \rightarrow \pi^+\pi^-J/\psi$  decay measured by Belle [5] suggests that the  $J^{PC} = 0^{++}$  and  $J^{PC} = 0^{+-}$  may be ruled out and strongly favors the  $J^{PC} = 1^{++}$  quantum numbers although the  $2^{++}$  combination cannot be excluded. A later analysis by CDF Collaboration [7] of the same decay is compatible with the Belle results and concludes from the dipion mass spectrum that the most likely quantum numbers should be  $J^{PC} = 1^{++}$  but cannot totally exclude the  $J^{PC} = 2^{-+}$  combination. These conclusions were confirmed by a new CDF analysis of the decay  $X(3872) \rightarrow \pi^+\pi^-J/\psi$  followed by  $J/\psi \rightarrow \mu^+\mu^-$  excluding all the other possible quantum numbers at 99.7 % confidence level [8]. However the small phase space available for the decay  $X(3872) \rightarrow D^0\bar{D}^0\pi^0$  observed by Belle [9] discards the  $J = 2$  leaving the  $1^{++}$  assignment as the most probable option.

In the  $1^{++}$  sector the only well established state in the PDG [10] is the  $\chi_{c_1}(1P)$  with a mass  $M = 3510.66 \pm 0.07\text{ MeV}$ . The first excitation is expected around  $3950\text{ MeV}$ . In this energy region Belle has reported the observation of three resonant structures denoted by  $X(3940)$ ,  $Y(3940)$  and  $Z(3930)$ . The last one was observed by Belle in the  $\gamma\gamma \rightarrow D\bar{D}$  reaction [11] and is already included in the PDG as the  $\chi_{c_2}(2P)$ . The  $X(3940)$  has been seen as a peak in the recoiling mass spectrum of  $J/\psi$  produced in  $e^+e^-$  collision. Its main decay channel is  $DD^*$  [12]. The  $Y(3940)$  appears as a threshold enhancement in the  $J/\psi\omega$  invariant mass distribution of the  $B \rightarrow J/\psi\omega K$  decay [13]. Finally, CDF Collaboration has reported a new structure in the  $B^+ \rightarrow J/\psi\phi K^+$  decay at a mass of  $4143 \pm 2.9 \pm 1.2\text{ MeV}$  [14]. This state, called  $Y(4140)$  has some similarities to the  $Y(3940)$  state as far as decay channel is concerned.

The decays measured for the  $X(3872)$  outlines a puzzling structure. In one hand the decays into  $\pi\pi J/\psi$  and  $\pi\pi\pi J/\psi$  through  $\rho$  and  $\omega$  mesons respectively suggests a sizable isospin breaking, incompatible with a  $c\bar{c}$  structure. In the other hand the radiative decays into  $\gamma J/\psi$  and  $\gamma\psi'$  suggest a sizable  $c\bar{c}$  component.

## COUPLED CHANNEL CALCULATION

In this work we present a coupled channel calculation of the  $1^{++}$  sector including both  $c\bar{c}$  and  $DD^*$  states. The calculation is done in the framework of the constituent quark model of Ref. [15] widely used in hadronic spectroscopy.

We start assuming a wave function given by

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi_{\alpha}\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M_1} \phi_{M_2} \beta\rangle \quad (1)$$

where  $|\psi_{\alpha}\rangle$  are  $c\bar{c}$  eigenstates of the two body Hamiltonian,  $|\phi_{M_i}\rangle$  are  $c\bar{n}$  ( $\bar{c}n$ ) eigenstates describing the  $D$  ( $\bar{D}$ ) mesons,  $|\phi_{M_1} \phi_{M_2} \beta\rangle$  is the two meson state with  $\beta$  quantum numbers coupled to total  $J^{PC}$  quantum numbers and  $\chi_{\beta}(P)$  is the relative wave function between the two mesons in the molecule. The eigenstates of the  $C$ -parity operator are given by  $DD^* \equiv D\bar{D}^* \pm \bar{D}D^*$ .

We use a phenomenological  $^3P_0$  model [16] to couple the two and four quark systems using the operator

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p+p') \left[ \mathcal{B}_1 \left( \frac{p-p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right]^{C=1, I=0, S=1, J=0} \quad (2)$$

where  $\mu$  ( $\nu = \bar{\mu}$ ) are the quark (antiquark) quantum numbers and  $\gamma' = 2^{5/2}\pi^{1/2}\gamma$  and  $\gamma = \frac{g}{2m}$  are dimensionless constants that gives the strength of the  $q\bar{q}$  pair creation from the vacuum. The value of gamma is fitted to the  $\psi(3770) \rightarrow DD$  decay width.

Using the wave-function from Eq. (1) with the two body eigenfunctions for all the mesons we apply the Resonant Group Method to obtain the dynamics in the two meson sector. We finally end up with the coupled channel equation

$$\sum_{\beta} \int \left( H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}^{eff}(P', P) \right) \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P') \quad (3)$$

where we include the  $^3S_1$  and  $^3D_1$   $DD^*$  partial waves and

$$V_{\beta'\beta}^{eff}(P', P) = \sum_{\alpha} \frac{V_{\beta'\alpha}(P') V_{\alpha\beta}(P)}{E - M_{\alpha}} \quad (4)$$

is an effective interaction between the two mesons due to the coupling with intermediate  $c\bar{c}$  states with

$$\langle \phi_{M_1} \phi_{M_2} \beta | T | \psi_{\alpha} \rangle = P V_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{\text{cm}}). \quad (5)$$

The  $c\bar{c}$  probabilities are given by

$$c_{\alpha} = \frac{1}{E - M_{\alpha}} \sum_{\beta} \int V_{\alpha\beta}(P) \chi_{\beta}(P) P^2 dP \quad (6)$$

with the normalization condition  $1 = \sum_{\alpha} |c_{\alpha}|^2 + \sum_{\beta} \langle \chi_{\beta} | \chi_{\beta} \rangle$ .

The results are given in Table 1. Part A corresponds to the isospin symmetric case while part B shows the effect of isospin breaking in phase space. In both cases no bound state is found without coupling to the  $c\bar{c}$  sector, neither in the  $I = 0$  nor in the  $I = 1$ . This coupling generates a new state with an energy close to the  $DD^*$  threshold.

Having in mind that the  $^3P_0$  model is probably too naive and we might be overestimating the value of  $\gamma$ , we vary this parameter to get the experimental binding energy. The probabilities are given in part C of Table 1.

## COMPARISON WITH THE DATA

In order to compare the predictions of our model with the experimental data we use a Flatté-like parametrization of the  $DD^*$  amplitude following Ref. [17, 18]. The differential cross section to final  $DD^*$  states is given by

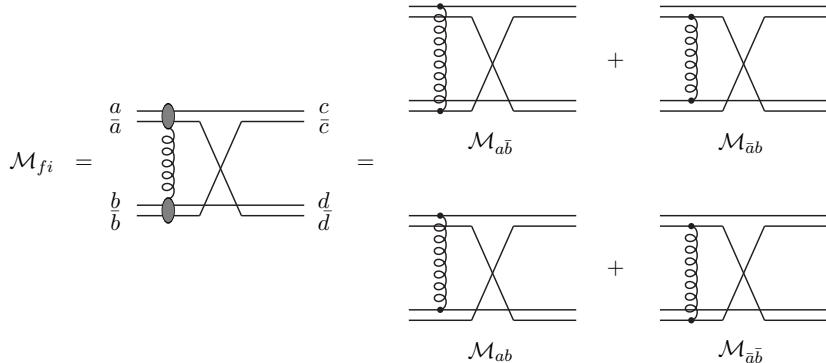
$$\frac{dBr(B \rightarrow KD^0 D^{*0})}{dE} = \mathcal{B} \frac{1}{2\pi} \frac{\Gamma_{D^0 D^{*0}}(E)}{|D(E)|^2} \quad (7)$$

where  $\mathcal{B}$  gives the branching to  $B \rightarrow KX(3872)$ ,  $\Gamma_{D^0 D^{*0}}(E)$  is the width calculated from the  $^3P_0$  model and

$$D(E) = E - E_f + \frac{i}{2}(\Gamma_{D^0 D^{*0}} + \Gamma_{D^+ D^{*-}} + \Gamma(E)) + \mathcal{O}(4\mu^2 \epsilon / \Lambda^2). \quad (8)$$

**TABLE 1.** Masses and channel probabilities for the three states in three different calculations. The first three states are found when we perform an isospin symmetric calculation with a value of  $\gamma$  fit to the decay  $\psi(3770) \rightarrow DD$ . The second three states show the effect of isospin breaking in the  $DD^*$  masses. The last three states correspond to a value of  $\gamma = 0.19$  that fits the experimental mass of the  $X(3872)$ . The probability is shown as zero when it is less than 0.5 %.

	$M (MeV)$	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$
A	3936	0 %	79 %	10.5 %	10.5 %
	3865	1 %	32 %	33.5 %	33.5 %
	3467	95 %	0 %	2.5 %	2.5 %
B	3937	0 %	79 %	7 %	14 %
	3863	1 %	30 %	46 %	23 %
	3467	95 %	0 %	2.5 %	2.5 %
C	3942	0 %	88 %	4 %	8 %
	3871	0 %	7 %	83 %	10 %
	3484	97 %	0 %	1.5 %	1.5 %



**FIGURE 1.** Diagrams included in the quark rearrangement process  $DD^* \rightarrow \rho J/\psi$ .

where  $\Gamma(E)$  accounts for the width due to other processes different from the opening of the near  $DD^*$  threshold.

The analysis of the  $B \rightarrow KX(3872) \rightarrow K\pi^+\pi^-J/\psi$  data is more involved because we have to calculate the  $DD^* \rightarrow \pi^+\pi^-J/\psi$  transition amplitude.

This can consistently be done in our formalism assuming that the process takes place through the  $DD^*$  components of the  $X(3872)$  which decays into  $\rho J/\psi$  and then into the final  $\pi^+\pi^-J/\psi$  state. The decay width of the process is given by

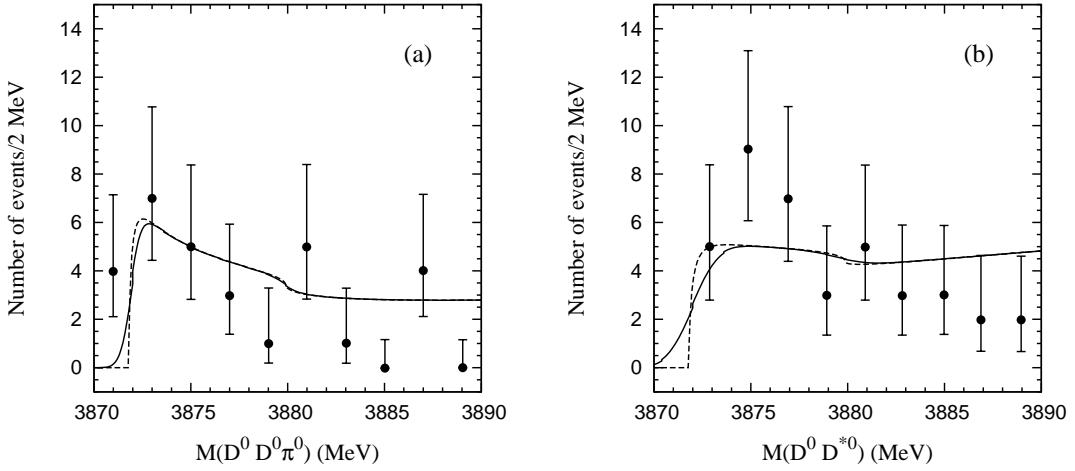
$$\Gamma_{\pi^+\pi^-J/\psi} = \sum_{JL} \int_0^{k_{max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\psi})^2 + \frac{\Gamma_\rho^2}{4}} \left| \mathcal{M}_{X \rightarrow \rho J/\psi}^{JL}(k) \right|^2. \quad (9)$$

The amplitude  $\mathcal{M}_{X \rightarrow \rho J/\psi}^{JL}$  is calculated in our model by the rearrangement diagrams of Fig. 1, averaged with the  $DD^*$  component of the  $X(3872)$  wave function. The rearrangement diagrams are calculated following Ref. [19]. The amplitude is given by

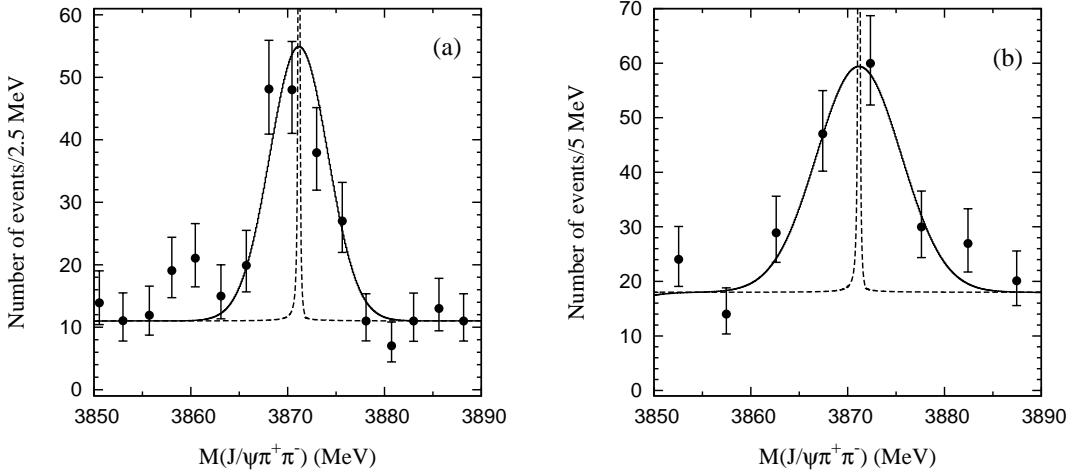
$$\mathcal{M}_{fi} = \sum_{i=a,\bar{a};j=b,\bar{b}} \mathcal{M}_{ij} \quad (10)$$

where

$$\mathcal{M}_{ij}(\vec{P}', \vec{P}) = \langle \phi_{M'_1} \phi_{M'_2} | H_{ij}^O | \phi_{M_1} \phi_{M_2} \rangle \langle \xi_{M'_1 M'_2}^{SFC} | \mathcal{O}_{ij}^{SFC} | \xi_{M_1 M_2}^{SFC} \rangle \quad (11)$$



**FIGURE 2.** Number of events for the decay  $B \rightarrow KD^0 D^0 \pi^0$  measured by Belle (a) and for the decay  $B \rightarrow KD^0 D^{*0}$  measured by BaBar (b). The solid and dashed lines shows the results from our model with and without the resolution functions as explained in the text.



**FIGURE 3.** Number of events for the decay  $B \rightarrow K\pi^+\pi^- J/\psi$  measured by Belle (a) and by BaBar (b). The solid and dashed lines shows the results from our model with and without the resolution functions as explained in the text.

The spin-flavor-color matrix elements are taken from Ref. [19].

Once the decay width  $\Gamma_{\pi^+\pi^-J/\psi}$  is calculated, the differential rate is given by

$$\frac{dBr(B \rightarrow K\pi^+\pi^-J/\psi)}{dE} = \mathcal{B} \frac{1}{2\pi} \frac{\Gamma_{\pi^+\pi^-J/\psi}(E)}{|D(E)|^2}. \quad (12)$$

In order to compare with the experimental data we determine the number of events distributions from the differential cross section following Ref. [18]. In all reactions a background is taken into account modelled as in Ref. [18]. For the  $B \rightarrow KD^0\bar{D}^0\pi^0$  the  $DD^{*0}$  signal interferes with the background and so a phase  $\phi^{Belle} = 0^0$  and  $\phi^{BaBar} = 324^0$  have been introduced. Also the experimental branching ratio  $B(D^{*0} \rightarrow D^0\pi^0) = 0.62$  is introduced. We use a value for  $\mathcal{B} = 3.5 \cdot 10^{-4}$  which is in the order of the one used in Ref. [18].

In Fig. 2 we compare our results with the  $B \rightarrow KD^0\bar{D}^0\pi^0$  data from Belle (a) and  $B \rightarrow KD^0\bar{D}^{*0}$  data from BaBar (b). The same comparison is done in Fig. 3 for the  $B \rightarrow K\pi^+\pi^-J/\psi$  data from Belle (a) and BaBar (b). In all figures

the dashed lines shows the results without resolution functions. The solid line gives the result using the resolution functions as in Ref. [18]. All the resolution functions are those given by Belle [20] and BaBar [21] collaboration with the exception of the BaBar  $D^0 D^{*0}$  resolution where we use the prescription from Ref. [18].

We find a good description of the Belle  $B \rightarrow K D^0 D^0 \pi^0$  data whereas the agreement is poor in the case of the BaBar data. It is important to notice that in the Belle analysis the mass of the  $X$  appears as  $3872 \text{ MeV}$  while in the BaBar data the resonance is located  $3 \text{ MeV}$  above. The BaBar mass value does not coincide with the mass of the  $X$  obtained in our calculation which may be the reason for the disagreement.

The  $B \rightarrow K \pi^+ \pi^- J/\psi$  data are equally well described for the Belle and BaBar experiments. In this case both Collaborations give similar values for the mass of the resonance, namely  $3871.4 \text{ MeV}$ , which are in much better agreement with our result.

Concerning other  $XYZ$  states, the  $X(3940)$ , decaying to  $D\bar{D}^*$ , with mass  $M = 3942 \pm 9 \text{ MeV}$  is a good candidate to our state with a 88%  $1^{++}$   $c\bar{c}$  component and mass  $M = 3942 \text{ MeV}$  (see Table 1 part C). With respect to the other two states, it has been suggested that both, the  $Y(3940)$  decaying to  $J/\psi \omega$  and  $Y(4140)$  decaying to  $J/\psi \phi$ , are  $D^* D^*$  and  $D_s^* D_s^*$  hadronic molecules with  $J^{PC} = 0^{++}$  or  $2^{++}$  respectively. We have explored these channels but we have not found in principle any molecular bound state.

As a summary, we have shown that the  $X(3872)$  emerges in a constituent quark model calculation as a dynamically generated mixed state of a  $DD^*$  molecule and  $\chi_{c1}(2P)$ . This structure allows to understand simultaneously the isospin violation showed by the experimental data and the radiative decay rates. Furthermore, we have demonstrated that this solution explains the new Belle data in the  $D^0 D^0 \pi^0$  and  $\pi^+ \pi^- J/\psi$  decay modes and the  $\pi^+ \pi^- J/\psi$  BaBar data. The original  $\chi_{c1}(2P)$  state acquires a significant  $DD^*$  component and can be identified with the  $X(3940)$ .

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